processes, but the isotherm is not possible; 3) the polytropic exponent at the throat is very nearly equal to the local isentropic exponent, and consequently the local Mach number at the throat is very nearly equal to one; and 4) there is a possibility of having almost an isobaric case in certain nozzles in which case the static temperature must change in the same ratio as the cross section.

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Solar Pressure Induced Librations of Spinning Axisymmetric Satellites

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Nomenclature

C_p = center of pressure of the satellite
I = inertia parameter, I_x/I_y
I_x, I_y, I_z = principal moments of inertia of the satellite
NN' = line of nodes
P = pericenter
R = distance between the satellite center of mass and the center of force
R_p = distance between the pericenter and the center of force
e = eccentricity
h_α = constant of the motion, Eq. (2)
i = inclination of the orbital plane with the ecliptic
$\bar{i}, \bar{j}, \bar{k}$ = unit vectors along x, y, and z axes, respectively
l, r = length and radius of satellite, respectively
p_o = solar radiation pressure
$\bar{u}$ = unit vector in the direction of the sun, $u_i \bar{i} + u_j \bar{j} + u_k \bar{k}$
u_i = $-\sin\phi \sin i \cos\beta \cos\gamma + (\sin\phi \cos i \sin\omega + \cos\phi \cos\omega)(\cos\beta \sin\gamma \cos\theta + \sin\beta \sin\theta) + (\sin\phi \cos i \cos\omega - \cos\phi \sin\omega)(\cos\beta \sin\gamma \sin\theta - \sin\beta \cos\theta)$
u_j = $\sin\phi \sin i \sin\gamma + (\sin\phi \cos i \sin\omega + \cos\phi \cos\omega) \cos\gamma \cos\theta + (\sin\phi \cos i \cos\omega - \cos\phi \sin\omega) \cos\gamma \sin\theta$
u_k = $-\sin\phi \sin i \sin\beta \cos\gamma + (\sin\phi \cos i \sin\omega + \cos\phi \cos\omega)(\sin\beta \sin\gamma \cos\theta - \cos\beta \sin\theta) + (\sin\phi \cos i \cos\omega - \cos\phi \sin\omega)(\sin\beta \sin\gamma \sin\theta + \cos\beta \cos\theta)$
x_o, y_o, z_o = rotating coordinate system with x_o normal to the orbital plane and y_o along the local vertical
x_1, y_1, z_1, x_2, y_2, z_2 = intermediate body coordinates resulting from rotations λ and β about z_0 and y_1 axes, respectively, Fig. 1
Φ = angle between the satellite axis of symmetry and the orbit normal
ε = distance between the center of pressure and the center of mass of the satellite
θ = orbital angle
μ = gravitational constant
ρ, τ = reflectivity and transmissibility of the satellite surface, respectively
ϕ = solar aspect angle
ω = angle between the line of apsides and the line of nodes

Superscript
$\dot{}$ = differentiation with respect to time
$'$ = differentiation with respect to θ

Subscript
o = initial condition

Introduction

INFLUENCE of the solar radiation pressure on the librational motion of spinning satellites has remained virtually unexplored. On the other hand, the importance of such a study becomes apparent when one recognizes the fact that the majority of the communications, applied technology, and scientific satellites are indeed spin stabilized and frequently negotiate high altitude orbits where the solar radiations constitute the major environmental force.

This Note investigates the coupled librational dynamics of an axisymmetric, cylindrical, slowly spinning satellite under the influence of the solar radiation pressure and gravity gradient torques. The derivation of the equations of motion and generalized forces for the satellite librating in an arbitrary orbit, being lengthy and routine, is omitted. However, the reference is cited for details. The nonlinear, nonautonomous, coupled equations are analyzed numerically and the response data presented as functions of the system parameters. Finally, the available information is condensed in the form of design plots which clearly emphasize the destabilizing influence of the radiation torques, characterized by the solar parameter.

Equations of Motion

Figure 1 shows an axisymmetric cylindrical satellite with the center of mass S moving in a Keplerian orbit about the center of force O. The spatial orientation of the axis of symmetry of the satellite is completely specified by two successive rotations γ and β, referred to as roll and pitch, respectively, which define the attitude of the satellite principal axes x, y, z, with respect to the inertial reference frame x', y', z'. The satellite spins in the x, y, z reference with angular velocity $\dot{\alpha}$. In terms of these modified Eulerian rotations, the governing equations of motion can be written as[1]:

$$(d/dt)(\dot{\alpha} - \dot{\gamma}\sin\beta + \dot{\theta}\cos\beta\cos\gamma) = Q_\alpha/I_x \tag{1a}$$

$$(d/dt)(\dot{\beta} - \dot{\theta}\sin\gamma) + (\dot{\gamma}\cos\beta + \dot{\theta}\sin\beta\cos\gamma)\{I(\dot{\alpha} - \dot{\gamma}\sin\beta + \dot{\theta}\cos\beta\cos\gamma) + (\dot{\gamma}\sin\beta - \dot{\theta}\cos\beta\cos\gamma)\} - 3(\mu/R^3)(I-1)\sin^2\gamma\sin\beta\cos\beta = Q_\beta/I_y \tag{1b}$$

$$d/dt\{-I(\dot{\alpha} - \dot{\gamma}\sin\beta + \dot{\theta}\cos\beta\cos\gamma)\sin\beta + (\dot{\gamma}\cos\beta + \dot{\theta}\sin\beta\cos\gamma)\cos\beta\} + I(\dot{\alpha} - \dot{\gamma}\sin\beta + \dot{\theta}\cos\beta\cos\gamma)\dot{\theta}\cos\beta\sin\gamma + (\dot{\beta} - \dot{\theta}\sin\gamma)\dot{\theta}\cos\gamma + (\dot{\gamma}\cos\beta + \dot{\theta}\sin\beta\cos\gamma)\dot{\theta}\sin\beta\sin\gamma + 3(\mu/R^3)(I-1)\sin\gamma\cos\gamma\cos^2\beta = Q_\gamma/I_y \tag{1c}$$

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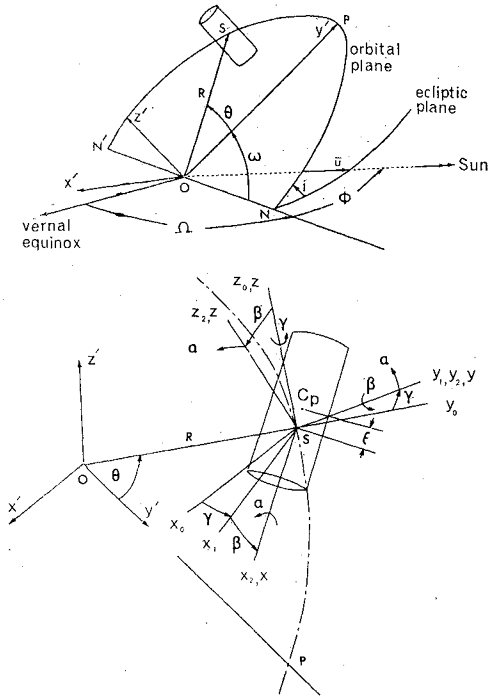

Fig. 1 Geometry of motion.

with the generalized forces in the α, β, and γ degrees of freedom given by

$$Q_\alpha = 0$$
$$Q_\beta = 2(1-\tau+\rho/3)p_0 r l \varepsilon u_k[(u_j^2+u_k^2)^{1/2} + (\pi r/2l)\{(1-\tau-\rho)/(1-\tau+\rho/3)\}|u_i|]$$
$$Q_\gamma = -2(1-\tau+\rho/3)p_0 r l \varepsilon u_j[(u_j^2+u_k^2)^{1/2} + (\pi r/2l)\{(1-\tau-\rho)/(1-\tau+\rho/3)\}|u_i|]\cos\beta$$

The generalized force in the α degree of freedom being zero, a first integral of motion defining the satellite spin rate $\dot{\alpha}$ is furnished by Eq. (1a),

$$\dot{\alpha} - \dot{\gamma}\sin\beta + \dot{\theta}\cos\beta\cos\gamma = h_\alpha \tag{2}$$

As h_α is a measure of the rotor spin, a dimensionless spin parameter σ, defined as

$$\sigma = (\dot{\alpha}/\dot{\theta})|_{\theta=\beta=\gamma=0} = (h_\alpha/\dot{\theta})|_{\theta=0} - 1$$

may be used to eliminate the cyclic coordinate α. Changing the independent variable to θ through the Keplerian orbital relations and making use of the spin parameter σ, the governing equations of motion in the pitch and roll degrees of freedom (Eqs. 1b and 1c, respectively) transform to

$$\beta'' - \gamma'\cos\gamma + [I(\sigma+1)\{(1+e)/(1+e\cos\theta)\}^2 + (\gamma'\sin\beta - \cos\beta\cos\gamma)](\gamma'\cos\beta + \cos\gamma\sin\beta) - \{3(I-1)/(1+e\cos\theta)\}\sin^2\gamma\sin\beta\cos\beta - \{2e\sin\theta/(1+e\cos\theta)\}(\beta' - \sin\gamma) = \{(1+e)^3/(1+e\cos\theta)^4\}Cu_k\{(u_j^2+u_k^2)^{1/2} + G|u_i|\} \tag{3a}$$

$$\gamma'' - 2\beta'\gamma'\tan\beta + 2\beta'\cos\gamma - I(\sigma+1)\{(1+e)/(1+e\cos\theta)\}^2(\beta' - \sin\gamma)\sec\beta + \{3(I-1)/(1+e\cos\theta) - 1\}\sin\gamma\cos\gamma - \{2e\sin\theta/(1+e\cos\theta)\}(\gamma' + \cos\gamma\tan\beta) = -\{(1+e)^3/(1+e\cos\theta)^4\}Cu_j\{(u_j^2+u_k^2)^{1/2} + G|u_i|\}\sec\beta \tag{3b}$$

where the solar parameter C and the solar aspect ratio G are defined as

$$C = (2R_p^3/\mu I_y)p_0 r l \varepsilon(1-\tau+\rho/3)$$
$$G = (\pi r/2l)(1-\tau-\rho)/(1-\tau+\rho/3)$$

These highly nonlinear, nonautonomous coupled equations of motion do not possess any known closed form solution. One is, therefore, forced to resort to a numerical approach to gain some appreciation as to the system behavior.

Analysis and Discussion of Results

The librational response of the system was studied by numerically integrating the equations of motion (3). The Adams-Bashforth predictor-corrector quadrature with the Runge-Kutta starter was used, in conjunction with a step size of 3°, which gave results of sufficient accuracy without involving excessive computational effort. The important system parameters were varied gradually over the range of interest and the system performance evaluated, both in circular and elliptic orbits with arbitrary orientations of the oribital plane. To isolate and emphasize the influence of the radiation pressure, no other disturbances in the form of initial conditions were introduced. The amount of information thus generated is rather extensive. However, for conciseness, only the typical results sufficient to establish trends are presented here.

Figure 2a shows the effect of the satellite inertia parameter I on the coning amplitude Φ_{max} and the average "nodding" frequency of the axis of symmetry, ω_n, expressed as oscillations per orbit, for different values of the solar parameter C. It may be observed that a satellite, when the solar pressure effects are neglected, remains in the equilibrium position ($\Phi=0$). However, with nonzero C (say, $C=0.05$), the librational motion is excited which increases in amplitude with increasing C. It is of interest to recognize the presence of a critical value of $I=1$ leading to large amplitude motion finally resulting in instability. This amplitude build-up was found to occur even for very small values of the solar parameter C, thus indicating a resonant behavior at these critical combinations of the system parameters. On the other hand, the

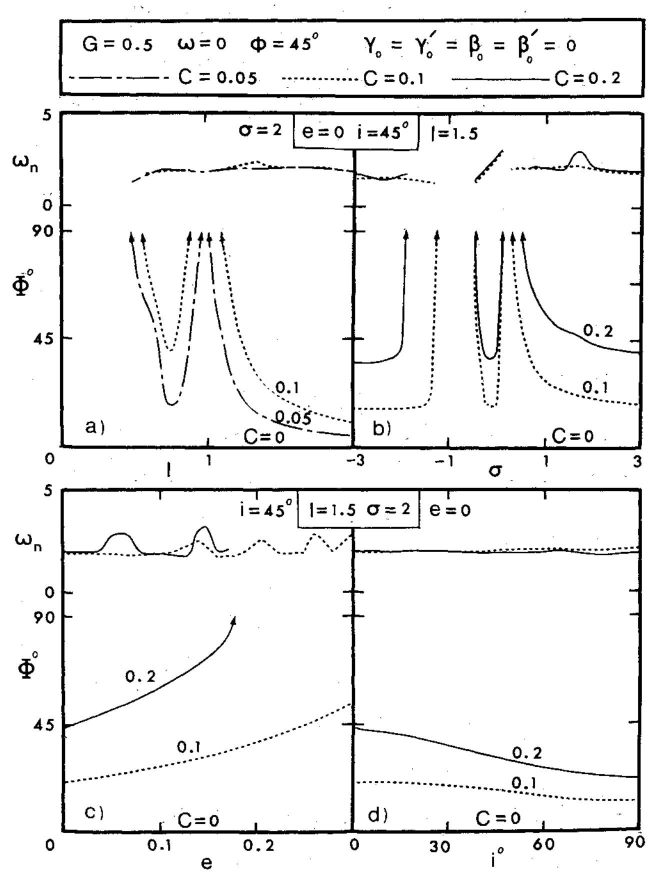

Fig. 2 System plots showing the coning angle and average nodding frequency as affected by a) inertial parameter; b) spin parameter; c) orbit eccentricity; d) orbit inclination.

average nodding frequency of the axis of symmetry appears to be relatively unaffected by changes in the inertia or solar parameter.

The effect of the spin parameter σ on the librational behavior is indicated in Fig. 2b. Here again, critical values of the spin parameter exist for which the satellite tumbles over. A large value of the spin parameter, in general, leads to smaller coning angles as anticipated.

Figure 2c shows the influence of the orbital eccentricity on the attitude motion. In general, higher values of the orbit eccentricity result in larger amplitude motion. Unlike the effect of the inertia and the spin parameters, no resonant behavior is noticed for the typical system parameters considered here.

The influence of the significant orbital parameters, such as i, ϕ, ω, and the solar aspect ratio G, on the satellite performance was also investigated. The amplitude of oscillation was found to reduce gradually with an increase in the orbital inclination from the ecliptic (Fig. 2d). Changes in the solar aspect angle ϕ, which depends upon the location of the line of nodes and the apparent position of the sun, did not affect the amplitude of librations and their frequency. The influence of the perigee position ω and the solar aspect ratio G was also found to be insignificant.

From design considerations, it would be desirable to assess the magnitude of the solar pressure torque that a satellite can withstand without exceeding the permissible bound of libration as governed by the mission requirements. This bound then would establish a criterion for stability. Here, the stability limit is purposely taken as a large value of $\Phi = \pi/2$ to emphasize the vulnerability of the satellite's performance to the solar pressure torque.

Figure 3a shows a typical stability chart for librational motion in a circular orbit with the solar radiation pressure as the only excitation. The equations of motion (3) were integrated over 15–20 orbits for a range of values of satellite inertia and spin parameters. The resulting information about the maximum amplitude of the coning angle Φ_{max} was then condensed in the form of the stability plots in the $I-\sigma$ space. The analysis shows that in addition to the main stable region for high inertia parameter values, there also exist small isolated stable areas. However, there are substantial unstable regions even for positive spin or large inertia parameters. It is observed that the stable areas reduce drastically, as expected, with an increase in the value of the solar parameter.

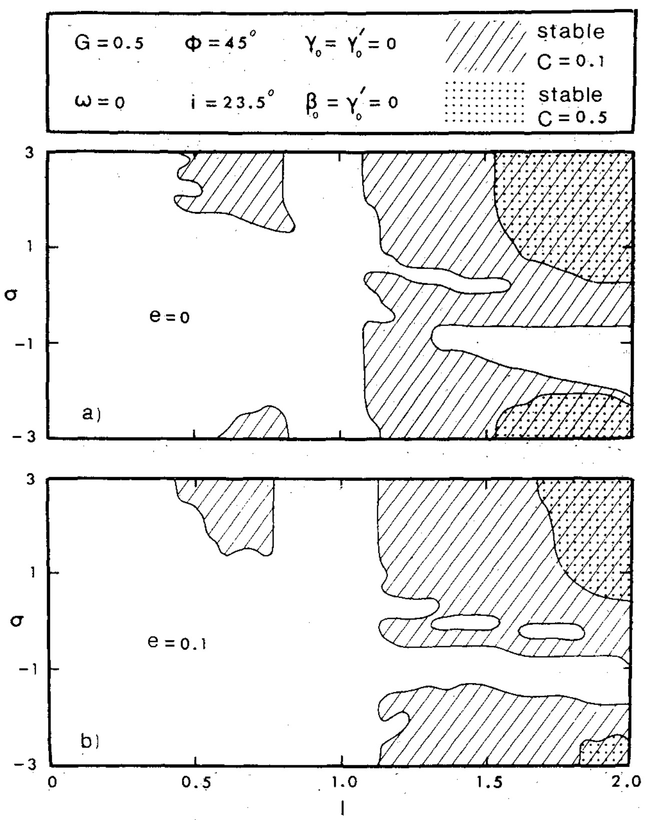

Fig. 3 Typical stability charts showing adverse influence of the solar radiation pressure. a) $e = 0$; b) $e = 0.1$.

The effect of the orbital eccentricity on the stability of librational motion is presented in Fig. 3b. An increase in orbital eccentricity further enhances the destabilizing influence of the solar pressure. The effect appears to be more pronounced for satellites spinning in a direction opposite to that of the oribital motion.

Of particular significance is the conclusion that the value of C as small as 0.5, which would physically correspond to $\varepsilon = 0.1$ ft for INTELSAT IV category of satellites ($I \simeq 0.7$) at synchronous altitude, causes the satellite to tumble over. The critical values of eccentricity, inertia, and spin parameters would only accentuate this behavior. Of course, in actual practice, a higher spin rate and/or active control system would counter this tendency. Nevertheless, the analysis clearly brings out the fact that the solar parameter C is of the same importance as I, σ, and e in the design of the satellite attitude control system.

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Example of Dynamic Interference Effects between Two Oscillating Vehicles

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Nomenclature

C_m = (pitching moment with static interference)/(qSl)
$C_{m\theta}$ = $\partial C_m/\partial\theta$, static pitching moment derivative
$C_{m\dot\theta}$ = $\partial C_m/\partial(l\dot\theta/2V)$, pitch damping derivative
$C_{m\theta}'$, $C_{m\dot\theta}'$ = effective derivatives, with dynamic interference
G, H = dynamic interference factors (for synchronous oscillation $G = C_{m\theta}'/C_{m\theta}$ and $H = C_{m\dot\theta}'/C_{m\dot\theta}$)
i = difference between the incidence of the orbiter and that of the booster
I_{YY} = moment of inertia about pitch axis
= reference length of each vehicle
q = dynamic pressure
S = reference area of each vehicle
Δx, Δz = longitudinal and vertical separation, respectively, between the centers of gravity, see Fig. 3

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